PRESSURE AND VELOCITY DISTRIBUTION IN SLIDE JOURNAL BEARING GAP BY LAMINAR UNSTEADY LUBRICATION

Paweł Krasowski

Gdynia Maritime University, Faculty of Marine Engineering
Morska Street 81-87,81-225 Gdynia, Poland
tel.:+48 58 6901331, fax:+48 58 6901399
e-mail:pawkras@am.gdynia.pl

Abstract

This paper shows results of numerical solutions an modified Reynolds equations for laminar unsteady oil flow in slide journal bearing gap. Laminar unsteady oil flow is performed during periodic and aperiodic perturbations of bearing load or is caused by the changes of gap height in time. Above perturbations occur mostly during the starting and stopping of machine. During modelling crossbar bearing operations in combustion engines, bearing movement perturbations from engine vertical vibrations causes velocity flow perturbations of lubricating oil on the bearing race and on the bearing slider in the circumferential direction. This solution example applies to isothermal bearing model with infinity length. Lubricating oil used in this model has Newtonian properties and constant dynamic viscosity. Perturbations connected with unsteady lubricating oil velocity in the circumferential direction on the slide bearing and on the slider of bearing were taken into consideration. Results are presented in the dimensionless hydrodynamic pressure and velocity distribution diagrams. Received solutions were compared with the solution received by the stationary lubrication flow in the slide journal bearing, which were made with the same parameter assumption by constant dynamic oil viscosity. Isothermal bearing model is similar to friction node model by steady-state heat load conditions.

Keywords: *laminar unsteady lubrication, journal slide bearing, pressure, velocity perturbation*

1. Introduction

This article refer to the unsteady, laminar flows issue, in which modified Reynolds number Re^* is [4, 5] smaller or equal to 2. Laminar, unsteady oil flow is performed during periodic and unperiodic perturbations of bearing load or is caused by the changes of gap height in the time. Above perturbations occur mostly during the starting and stopping of machine. Lubricated oil disturbance velocity the pin and on the bearing shell was also consider in the article. Reynolds equation system describing Newtonian oil flow in the gap of transversal slide bearing was discussed in the articles [3, 4]. Velocity perturbations of oil lubricated flow on the pin can be caused by torsion pin vibrations during the rotary movement of the shaft. Perturbations are proportional to torsion vibration amplitude, frequent constraint and to pin radius of the shaft. Oil velocity perturbations on the shell surface can be caused by rotary vibration of the shell together with bearing casing. This movement can be consider as kinematical constraint for whole bearing friction node. Isothermal bearing model can be approximate to bearing operation in friction node under steady-state thermal load conditions for example bearing in generating set on ship.

2. Reynolds equation and hydrodynamic pressure

The unsteady, laminar and isotherm flow Newtonian oil in journal bearing gap is described for modified Reynolds equation [1, 2] from Newtonian oil with constant and variable dynamic viscosity depended for pressure. In considered model we assume small unsteady disturbances and in order to maintain the laminar flow, oil velocity V_i^* and pressure p_1^* are total of dependent quantities $\widetilde{V}_i$; $\widetilde{p}_1$ and independent quantities $V_i; p_1$ from time [3, 5] according to equation (1).

$$V_i^* = V_i + \widetilde{V}_i \;, \qquad p_1^* = p_1 + \widetilde{p}_1 \;, \;\; \mathrm{i} = 1,2,3. \tag{1}$$

Unsteady components of dimensionless oil velocity and pressure are [4] in following form of infinite series:

$$\widetilde{\mathrm{V}}_\mathrm{i}(\varphi, \mathrm{r}_1, \mathrm{z}_1, \mathrm{t}_1) = \sum_{\mathrm{k}=1}^{\infty} \mathrm{V}_\mathrm{i}^{(\mathrm{k})}(\varphi, \mathrm{r}_1, \mathrm{z}_1) \exp(\mathrm{jk}\omega_0 \mathrm{t}_0 \mathrm{t}_1), \; \mathrm{i}=1,2,3,$$

$$\widetilde{p}_1(\varphi, r_1, z_1, t_1) = \sum_{k=1}^{\infty} p_1^{(k)}(\varphi, r_1, z_1) \exp(jk\omega_0 t_0 t_1), \tag{2}$$

where:
ω_0 - angular velocity perturbations in unsteady flow,
j = $\sqrt{-1}$ imaginary unit.

Components of oil velocity V_φ, V_r, V_z in cylindrical co-ordinates r,φ,z have presented as V_1, V_2, V_3 in dimensionless [1] form:

$$V_\varphi = UV_1, \qquad V_r = \psi UV_2, \qquad V_z = \frac{U}{L_1} V_3, \tag{3}$$

where:
U - peripheral journal velocity U=ωR,
ω - angular journal velocity,
R - radius of the journal,
ψ - dimensionless radial clearance ($10^{-4} \le \psi \le 10^{-3}$),
2b - length of bearing,
L_1 - dimensionless bearing length,
ε - radial clearance:

$$\psi = \frac{\varepsilon}{R} \;; \qquad \mathrm{L}_1 = \frac{b}{R}. \tag{4}$$

Putting following quantities [1],[4]: dimensionless values density ρ_1, hydrodynamic pressure p_1^*, time t_1, longitudinal gap height h_1, radial co-ordinate r_1 and co-ordinate z_1.

$$\begin{array}{lll} \rho = \rho_0\, \rho_1, & p = p_0\, p_1, & \mathrm{t} = \mathrm{t}_0\, t_1 \\ \mathrm{z} = \mathrm{b}\, \mathrm{z}_1, & \mathrm{r} = R(1 + \psi \mathrm{r}_1), & \mathrm{h} = \varepsilon \mathrm{h}_1 \end{array}. \tag{5}$$

Rule of putting dimensionless velocity and pressure quantities in unsteady and steady part of the flow stays similar. Following symbols with bottom zero index signify density, dynamic viscosity, pressure and time describe characteristic dimension values assigned to adequate quantities. Reynolds number Re, modified Reynolds number Re^* and Strouhal Number St is in form [1]:

$$\mathrm{p}_0 = \frac{\mathrm{U}\eta_0}{\psi^2 R} \;; \quad \mathrm{Re} = \frac{U\,\rho_0\,\psi R}{\eta_0} \;; \quad \mathrm{Re}^* = \psi\,\mathrm{Re}\,; \quad \mathrm{St} = \frac{\varepsilon}{\mathrm{Ut}_0}. \tag{6}$$

In work [1, 3] are presented general equations for dimensionless components circumferential velocities $V_1^{(k)}$ and longitudinal velocities $V_3^{(k)}$ in form:

$$V_i^{(k)} = \frac{j}{\eta_1 A_k}\frac{\partial p_1^{(k)}}{\partial x_i}\left\{1-\frac{\exp\left(r_1\sqrt{jA_k}\right)+\exp\left[\left(h_1-r_1\right)\sqrt{jA_k}\right]}{1+\exp\left(h_1\sqrt{jA_k}\right)}\right\}+ \\ +\frac{1}{k^2}\left\{V_{i0}\frac{\exp\left[\left(r_1-h_1\right)\sqrt{jA_k}\right]-\exp\left[-\left(r_1-h_1\right)\sqrt{jA_k}\right]}{\exp\left(-h_1\sqrt{jA_k}\right)-\exp\left(h_1\sqrt{jA_k}\right)}+V_{ih}\frac{\exp\left(-r_1\sqrt{jA_k}\right)-\exp\left(r_1\sqrt{jA_k}\right)}{\exp\left(-h_1\sqrt{jA_k}\right)-\exp\left(h_1\sqrt{jA_k}\right)}\right\}, \tag{7}$$

where:

$$A_k = k\frac{\rho_1}{\eta_1}\mathrm{Re}\,St\;\omega_0 t_0 \qquad k=1;2;3.....,\ i=1,3,\ x_1=\varphi;\ x_3=z_1. \tag{8}$$

Dimensionless quantities V_{i0} , V_{ih} are components perturbations in circumferential velocities (i=1) and longitudinal (i=3) on the journal (r_1 = 0) and the sleeve (r_1 = h_1). Dimensionless components $V_1^{(k)}$ unsteady flow for circumference of oil velocity and $V_3^{(k)}$ for longitudinal of oil velocity we assume on the oscillating journal and sleeve surface the following boundary conditions:

$$\begin{cases} V_1^{(k)}(\varphi; r_1=0; z_1)=\gamma_V\dfrac{V_{10}}{k^2} \\ V_1^{(k)}(\varphi; r_1=h_1; z_1; t_1)=\gamma_V\dfrac{V_{1h}}{k^2} \end{cases}, \qquad \begin{cases} V_3^{(k)}(\varphi; r_1=0; z_1; t_1)=\gamma_V\dfrac{V_{30}}{k^2} \\ V_3^{(k)}(\varphi; r_1=h_1; z_1; t_1)=\gamma_V\dfrac{V_{3h}}{k^2} \end{cases}. \tag{9}$$

Quantity γ_V are factor of scale for velocity perturbations, dependent for acceptant of term series (2). Velocity perturbation U_0 in direction φ on the journal and velocity perturbation U_h on the sleeve, velocity perturbation V_0 in direction z_1 on the journal and V_h on the sleeve have following dimensionless form:

$$V_{10}\equiv\frac{U_0}{U},\qquad V_{1h}\equiv\frac{U_h}{U}, \\ V_{30}\equiv\frac{V_0}{L_1 U},\qquad V_{3h}\equiv\frac{V_h}{L_1 U}. \tag{10}$$

Presented for formulas (7) components velocity perturbations include derivatives components of pressure perturbation $p_1^{(k)}$ by coordinates φ and z_1. Reynolds equations [1, 3-4] for pressure components $p_1^{(k)}$ by laminar unsteady oil flow have form:

$$\frac{\partial}{\partial\varphi}\left(\frac{h_1^3}{\eta_1}\frac{\partial p_1^{(k)}}{\partial\varphi}\right)\cos(k\omega_0 t_0 t_1)+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left(\frac{h_1^3}{\eta_1}\frac{\partial p_1^{(k)}}{\partial z_1}\right)\cos(k\omega_0 t_0 t_1)=-12\gamma_h\frac{\omega_0}{\omega}\frac{h_1}{k}\sin(k\omega_0 t_0 t_1)+ \\ -\frac{6}{k^2}\gamma_V\left\{\frac{\partial}{\partial\varphi}\left[h_1\left(V_{10}+V_{1h}\right)\right]+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[h_1\left(V_{30}+V_{3h}\right)\right]-2\left(V_{1h}\frac{\partial h_1}{\partial\varphi}+\frac{1}{L_1^2}\frac{\partial h_1}{\partial z_1}\right)\right\}\cos(k\omega_0 t_0 t_1)+ \\ +\frac{1}{2k^2}\gamma_V\left\{\frac{\partial}{\partial\varphi}\left[A_k h_1^3\left(V_{10}+V_{1h}\right)\right]+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[A_k h_1^3\left(V_{30}+V_{3h}\right)\right]\right\}\sin(k\omega_0 t_0 t_1). \tag{11}$$

Reynolds equation describing total dimensionless pressure p_1^* (sum steady and unsteady components) in oil journal bearing gap [1] by unsteady, laminar, isotherm Newtonian flow along with disturbances of peripheral velocity V_{10} on the journal and V_{1h} on the sleeve and disturbances of velocity on journal length V_{30} on the journal and V_{3h} on the sleeve has following form:

$$\frac{\partial}{\partial\varphi}\left(h_1^3\frac{\partial p_1^*}{\partial\varphi}\right)+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left(h_1^3\frac{\partial p_1^*}{\partial z_1}\right)=6\frac{\partial h_1}{\partial\varphi}+$$
$$+\frac{1}{2}\frac{\rho_1}{\eta_1}\mathrm{Re}^* n\gamma_V\left\{\frac{\partial}{\partial\varphi}\left[h_1^3(V_{10}+V_{1h})\right]+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[h_1^3(V_{30}+V_{3h})\right]\right\}\sum_{k=1}^{\infty}A_{(k)}+ \tag{12}$$
$$-6\gamma_V\left\{\frac{\partial}{\partial\varphi}\left[h_1(V_{10}+V_{1h})\right]+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[h_1(V_{30}+V_{3h})\right]-2\left(V_{1h}\frac{\partial h_1}{\partial\varphi}+\frac{1}{L_1^2}V_{3h}\frac{\partial h_1}{\partial z_1}\right)\right\}\sum_{k=1}^{\infty}B_{(k)}$$

for $0\le\varphi\le\varphi_e$; $0\le r_1\le h_{p1}$; $-1\le z_1\le 1$; $0\le t_1\le t_k$; $p_1^*=p_1^*(\varphi;z_1;t_1)$

Sum for series $\sum_{k=1}^{\infty}A_k$ and $\sum_{k=1}^{\infty}B_k$ in right side of Reynolds equation (12) are results from conservation of the momentum solutions and were define in work [1, 2].

$$\sum_{k=1}^{\infty}A_{(k)}=\sum_{k=1}^{\infty}\frac{\sin(k\omega_0 t_0 t_1)}{k}=\begin{cases}\dfrac{\pi-\omega_0 t_0 t_1}{2} & 0<t_1<1\\ 0 & t_1=0;1\end{cases}$$
$$\sum_{k=1}^{\infty}B_{(k)}=\sum_{k=1}^{\infty}\frac{\cos(k\omega_0 t_0 t_1)}{k^2}=\frac{1}{4}\left[(\pi-\omega_0 t_0 t_1)^2-\frac{\pi^2}{3}\right]\quad 0\le t_1\le 1 \tag{13}$$

The equation solution (12) for bearing with infinity length determines unsteady dimensionless hydrodynamic pressure function $\tilde{p}_1$ in following [2] form:

$$\tilde{p}_1=\frac{1}{2}\rho_1\mathrm{Re}^* n\gamma_V(V_{10}+V_{1h})\left(\varphi-h_{1e}^3\int_0^{\varphi}\frac{d\varphi}{h_1^3}\right)\sum_{k=1}^{\infty}A_{(k)}-\gamma_V p_{10}(V_{10}-V_{1h})\sum_{k=1}^{\infty}B_{(k)}. \tag{14}$$

Pressure p_{10} is located in the oil gap by steady flow and by constant oil dynamic viscosity.

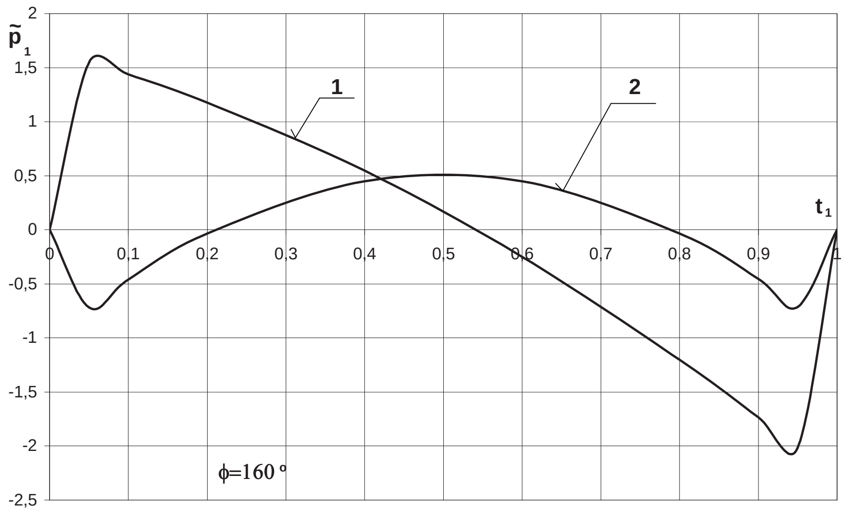


Fig. 1. Pressure distributions $\tilde{p}_1$ in place φ=160° in the time t_1 by velocity perturbations: 1) V_{10}=0,05; V_{1h}=0,025; 2) V_{10}=0,05; V_{1h}=-0,05

Pressure perturbation $\tilde{p}_1$ course in point φ=160° presented Fig. 1 by following circumferential velocity perturbations:

1. velocity perturbations on the journal V_{10}= 0,05 and on the sleeve V_{1h}=0,025.
2. velocity perturbations on the journal V_{10}= 0,05 and on the sleeve V_{1h}= -0,05.

Two velocities perturbations will analysed in this article.

3. Velocity distribution in bearing gap

We analyst cylindrical bearing infinite length with circumferential velocity perturbations on the journal V_{10} and on the sleeve V_{1h}. Circumferential velocity perturbations are caused by torsional vibrations shaft (on the journal) or circumferential displacement frame bearing (on the sleeve)In the further numerical analysis relation time t_0 was taken into account as a propagation period of axial velocity perturbation of lubricating oil. Express by a formula (7) are in form:

$$A_k = k\frac{\rho_1}{\eta_1}\mathrm{Re}\,\mathrm{St}\;\omega_0\,t_0 = k\frac{\rho_1}{\eta_1}\mathrm{Re}^*\,n \quad k = 1;2;3..... \quad . \tag{15}$$

In case where oil velocity perturbations are caused by forced vibrations of engine then the number n in equation (5) define multiplication of perturbation frequency ω_0 to angular velocity of engine crankshaft ω. Multiplication factor n is equal to number of cylinder c in two-stroke engine (s=2) or in four-stroke engine (s=4) to number of cylinders c/2 :

$$n = \frac{\omega_0}{\omega} = \begin{cases} c & s = 2 \\ \frac{c}{2} & s = 4 \end{cases}. \tag{16}$$

Solution Reynolds equation (11) for cylindrical bearing infinity length and Reynolds boundary conditions where film oils ended $\varphi=\varphi_e$ are in form:

$$\begin{aligned}\frac{\partial p_1^{(k)}}{\partial\varphi}\cos(k\omega_0 t_0 t_1) = &-\frac{6\eta_1}{k^2}\gamma_V(V_{10}-V_{1h})\frac{h_1-h_{1e}}{h_1^3}\cos(k\omega_0 t_0 t_1) + \\ &+\gamma_V\frac{\rho_1}{2k}\mathrm{Re}^*\,n(V_{10}+V_{1h})\frac{h_1^3-h_{1e}^3}{h_1^3}\sin(k\omega_0 t_0 t_1)\end{aligned}, \tag{17}$$

where: h_1 and h_{1e} height of gap

$$h_1(\varphi) = 1 + \lambda\cos\varphi, \qquad h_{1e} = h_1(\varphi_e). \tag{18}$$

Velocity circumferential perturbation (7) are in formula:

$$\begin{aligned}\tilde{V}_1 = &\frac{6\eta_1}{\rho_1\,\mathrm{Re}^*\,n}\gamma_V(V_{10}-V_{1h})\frac{h_1-h_{1e}}{h_1^3}\sum_{k=1}^{\infty}\frac{P(k)}{k^3}\cos(2k\pi t_1) + \\ &-\frac{\gamma_V}{2}(V_{10}+V_{1h})\frac{h_1^3-h_{1e}^3}{h_1^3}\sum_{k=1}^{\infty}\frac{P(k)}{k^2}\sin(2k\pi t_1) + \\ &+\gamma_V V_{10}\sum_{k=1}^{\infty}\frac{A(k)}{k^2}\cos(2k\pi t_1) + \gamma_V V_{1h}\sum_{k=1}^{\infty}\frac{B(k)}{k^2}\cos(2k\pi t_1)\end{aligned}. \tag{19}$$

Components of series P(k), A(k) i B(k) from formula (19) are in following forms:

$$
\begin{aligned}
P(k) &= \frac{\sin(sh_1X_k)\sinh[(1-s)h_1X_k]+\sin[(1-s)h_1X_k]\sinh(sh_1X_k)}{\cosh(sh_1X_k)+\cos(sh_1X_k)} \\
A(k) &= \frac{-\cosh(sh_1X_k)\cos[(2-s)h_1X_k]+\cosh[(2-s)h_1X_k]\cos(sh_1X_k)}{\cosh(2h_1X_k)-\cos(2h_1X_k)} \\
B(k) &= \frac{\cosh[(1+s)h_1X_k]\cos[(1-s)h_1X_k]-\cosh[(1-s)h_1X_k]\cos[(1+s)h_1X_k]}{\cosh(2h_1X_k)-\cos(2h_1X_k)} \\
s &= \frac{r_1}{h_1}, \qquad \gamma_V = \frac{6}{\pi^2}, \qquad X_k = \sqrt{k\frac{\rho_1}{2\eta_1}\mathrm{Re}^* n}, \qquad k = 1,2,3...
\end{aligned} \tag{20}
$$

Additional symbol, s, marks dimensionless parameter - height of gap ($0 \le s \le 1$). In numerical calculation example oil with constant density was assume, what is equivalent to quantity ρ_1. In presented calculation way an expression value is assumed $n\rho_1\mathrm{Re}^* = 12$, what is approximately

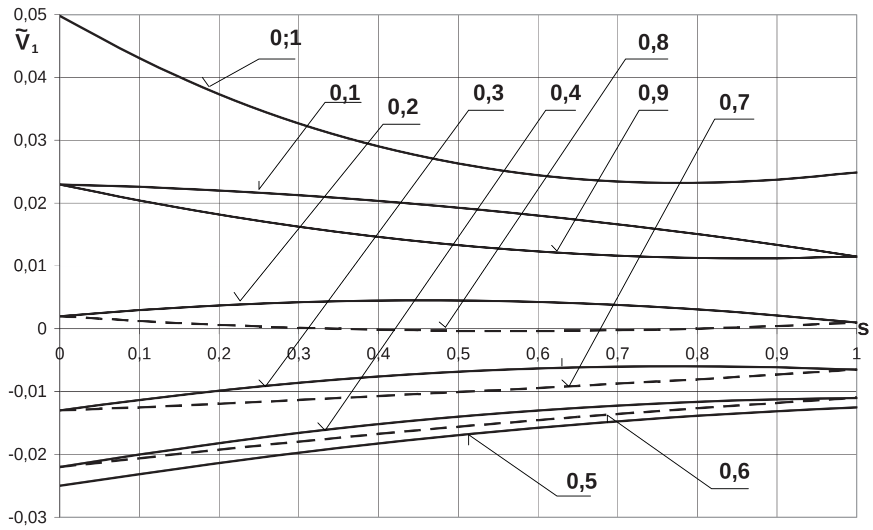


Fig. 2. Velocity distribution $\widetilde{V}_1$ in the circumferential direction, in gap height s, in the time t_l by velocity perturbations V_{10} = 0,05, V_{1h} = 0,025

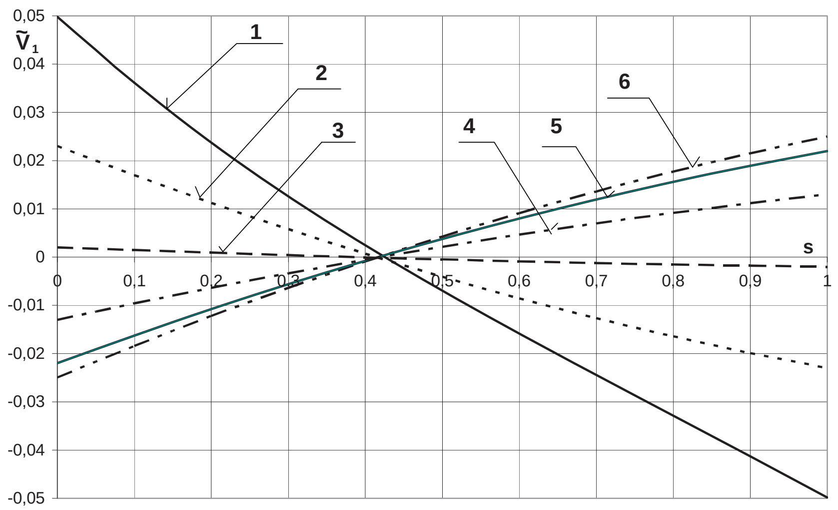


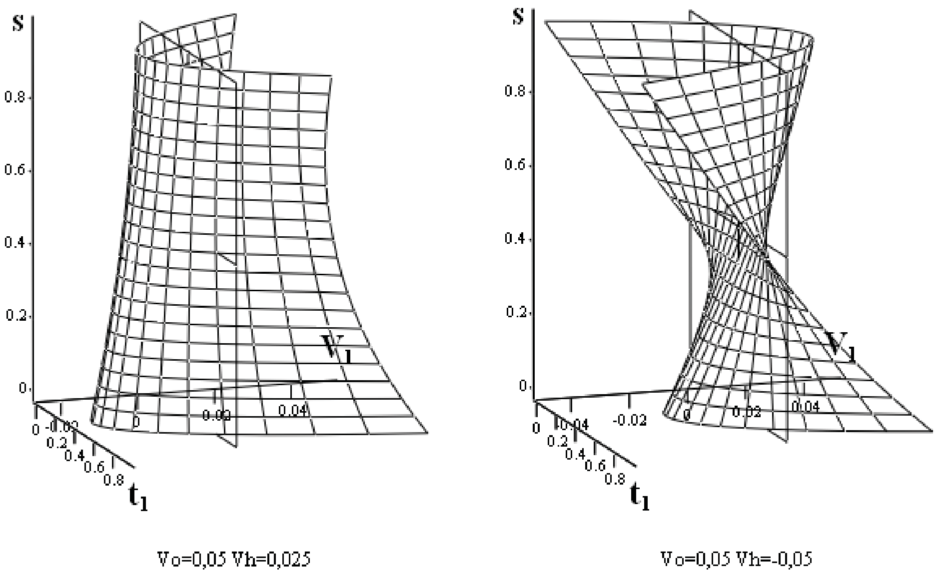
Fig. 3. Velocity distribution $\widetilde{V}_1$ in the circumferential direction by velocity perturbations V_{10} = 0.05, V_{1h} = - 0.05, in gap height s, in the time t_l : 1) 0; 1, 2) 0.1; 0.9 , 3) 0.2; 0.8 , 4) 0.3; 0.7 , 5) 0.4; 0.6 , 6) 0.5

equivalent to force over first frequency torsion vibrations force of six cylinder engine shaft. Examples apply to bearing with constant dependent eccentricity λ=0,6. Numerical examples velocity distributions are by following circumferential velocity perturbations:

1) V_{10}=0,05 , V_{1h}=0,025,
2) V_{10}=0,05 , V_{1h}=-0,05.

Determine a distributions of circumferential velocity in gap height s by gap crosswise section of coordinate φ=160°. The results of velocity distribution in the time t_1 show Fig. 2 and 3. Velocity perturbation are values different sign from initial perturbations V_{10}, V_{1h} and absolutely lesser.

Graphs velocity distributions from Fig. 2 are unsymmetrical in the time t_1 . Graphs velocity distributions from Fig. 3 are symmetrical in the time t_1. Causes are pressure distributions [2] in the time.



a b

Fig. 4. Velocity distribution $\widetilde{V}_1$ in the circumferential direction in gap height s in the time t_1, by velocity perturbations a)V_{10} = 0,05, V_{1h} = 0,025 ; b) V_{10} = 0,05, V_{1h} = -0,05

In Fig. 4 presented three-dimensional graphs distributions of velocity perturbation $\widetilde{V}_1$ for descript two case velocity perturbations in dimensionless gap height s (vertical axis) in the time t_1. Velocity perturbation are presented values for horizontal axis V_1. Additional in Fig. 3 mark frame zero plane, when perturbations of velocity are zero.

4. Conclusions

Presented solution of Reynolds Equation, for unsteady laminar Newtonian flow of lubricated oil, enables initial opinion to hydrodynamic pressure and velocity distribution as a basic slide bearing operating parameter. Unsteady velocity perturbation on the journal and sleeve effect on hydrodynamic pressure and velocity distribution in lubricated gap. Pressure variation in bearing have periodical character equal to periodical velocity perturbation time and this variations value and character depend on type of perturbation. Pressure increase and decrease and velocity distributions are not symmetrical during the perturbation time. Author is aware of simplifications that were assumed in presented model which apply to Newtonian oil and to isothermal bearing model. Despite that presented calculation example apply to bearing with infinity length, obtained conclusions can be useful to pressure and velocity distribution by laminar, unsteady lubrication of cylindrical slide bearing with finite length.

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